**QP Code: 22/PT/13/XB(ii)**

# POST-GRADUATE COURSE

## Term End Examination — June, 2022/December, 2022

## MATHEMATICS

### Paper-10B(ii) : MECHANICS OF SOLIDS

**( Applied Mathematics )**

**( Spl. Paper )**

Time : 2 hours ] [ Full Marks : 50

Weightage of Marks : 80%

**Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.**

**Use of scientific calculator is strictly prohibited.**

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

a) When will you call an elastic body to be in a state of plane strain ?

b) What is meant by the line of shear stress ?

c) Obtain the tensional rigidity of a rod of circular cross-section.

d) Define plane wave and write the form of a plane wave type disturbance propagating through the medium.

e) State the assumptions made in the small deflection theory of thin elastic plates.

f) Define the stress and strain deviators.

g) When is the biharmonic function $U = U(x_1, x_2)$ called Airy's stress function ?

h) What are the conditions of the clamped edge for transverse vibration of a thin rod ?

2. Show the problem of torsion of a long cylindrical rod of any cross-section of isotropic material twisted by a couple at one end, other end being fixed, is equivalent to the boundary value problem

$$\nabla_1^2\Phi = -2,\ (x_1, x_2)\in S,\ \nabla_1^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$$

and $\Phi$ = constant on the boundary $L$ of $S$, $S + L$ is a normal cross-section of the rod and $\Phi$ is Prandtl's stress function. 10

3. What are Rayleigh waves ? Show that the velocity $c$ with which Rayleigh waves propagate satisfies the equation

$$\left(2-\frac{c^2}{\beta^2}\right)^2 - 4\left(1-\frac{c^2}{\alpha^2}\right)^{1/2}\left(1-\frac{c^2}{\beta^2}\right)^{1/2} = 0$$

where the $\alpha$ and $\beta$ are to be defined by you. 10

4. Derive stress-strain relations of Von-Mises. Show that for incompressible materials the stress-strain relations are expressed in terms of the components of stress deviator. 10

5. State and prove the theorem of minimum potential energy. 10

6. Obtain the displacement components $u_1$ and $u_2$ in terms of two harmonic functions $\phi(z)$ and $\psi(z)$ for plane elastostatic problems. 10

7. Determine the magnitude of the force which gives rise to the displacements

$$u = B\frac{x}{r(z+r)},\ v = B\frac{y}{r(z+r)},\ w = B\frac{z}{r}$$

where $B$ is constant and $r^2 = x^2 + y^2 + z^2$,

in a hemisphere at the origin along the direction of the positive $z$-axis, the origin being at the centre of the hemisphere on the plane area. 10